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NUMERICAL MODELING OF THE APPEARANCE OF LOCALIZED STRAIN BANDS IN POROUS MATERIALS

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The capacity for deformation localization is an important characteristic feature of plastically deformable solids. It is of theoretical and practical interest to elucidate the circumstances and conditions of localization and to determine the parts of a sample in which localization can occur. Extensive literature is devoted to the localization of deformation. This paper concerns only the macroscopic manifestations of localization in the form of slip bands (Luders bands).

Most naturally these effects can be modeled within the framework of approaches that allow for both the strengthening and weakening of materials under deformation. The theory of plastic porous solids refers precisely to this class of models. An increase in porosity under stretching weakens the material. The localization condition in the simplest case reduces to the "violation of equilibrium" (in some sense) between strengthening and weakening toward weakening. Briefly this is discussed in the first section although the paper mainly reports results of direct numerical modeling of the appearance of slip bands in porous materials.

1. The Peculiarities of Deformation Localization in Porous Materials. A macroscopic slip band is a narrow region of active plastic strain dividing the practically nondeformed material on each side of the band. As a rule, one of the dimensions of this localized flow region is substantially smaller than the rest. However, as soon as one passes to the limiting case and considers a slip band as the velocity jump surface in the irreversibly deformed porous material, these jumps turn out to be nonexistent [1]. It is more convenient to regard the slip band as the limiting approach of two neighboring (e.g., at a given distance $h \ll 1$) surfaces of weak jumps (characteristics) without separating the development of localization from the final result. On the surfaces of weak jumps the velocity vector is continuous, whereas the components of the strain rate tensor are discontinuous. As $h \to 0$ we arrive, generally speaking, at the required surface of the velocity jump in the material studied.

We further consider only plane deformation of rigid-plastic porous material. The numerical analysis is based on the theory of plastic flow.

The slope angle β of the characteristics (if they exist) to that of the deformation plane axes for an irreversibly compressible plastic material whose loading surface is of the form

$$\Phi(p, \tau, \theta, \Gamma) = 0$$

is determined either from the relation [1]

$$\tan\beta = \frac{\sqrt{\Phi_\tau^2 - \frac{1}{3}\Phi_p^2}\sin 2\alpha \pm \sqrt{\Phi_\tau^2 - \frac{4}{3}\Phi_p^2}}{\Phi_p - \sqrt{\Phi_\tau^2 - \frac{1}{3}\Phi_p^2}\cos 2\alpha}$$

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or, in terms of the strain rates, from the relation [2]

$$\tan\beta = \frac{\sin 2\alpha \pm \sqrt{1 - s_1^2}}{\cos 2\alpha + s_1}.$$

In this case, p, τ are the first invariant tensor and the second invariant of the deviator stress tensor; $s_1 = e/\gamma_1$ ($\gamma_1 = \sqrt{(e_x - e_y)^2 + 4\gamma_{xy}^2}$); e and γ are analogous invariants of the strain rate tensors; α is the slope angle of one of the principal directions of either the strain rate tensors or of the stress tensor to the chosen coordinate axis; θ is the porosity; Γ is the strengthening parameter of a solid phase in the porous medium.

At least one family of characteristics exists providing that

$$2 \mid \Phi_p \mid \leqslant \sqrt{3} \mid \Phi_\tau \mid \tag{1.1}$$

or

$$s_1 \leqslant 1.$$

The above condition is necessary for the existence of a localization band. In this case, the band forms the angle β with the coordinate axis. We set Φ in the form [2]

$$\Phi = \frac{p^2}{\psi} + \frac{\tau^2}{\varphi} - (1 - \theta)\sigma_y^2,$$

where $\psi = (2/3)((1 - \theta)^3/\theta)$; $\varphi = (1 - \theta)^2$; σ_y is a constant that corresponds to the constant in the Mises yield condition for a nonporous material. In uniaxial stretching, the associative law gives

$$s_1 = \frac{e}{\gamma_1} = \frac{3\theta}{4 - 3\theta}. \tag{1.2}$$

As follows from Eq. (1.2), the slope angle of the slip band depends on porosity. At $\theta = 0$ we obtain the well-known result for the slope angle of the slip band in a nonporous material.

Since at the instant when the band appears the value of s_1 fits both the sample as a whole and the behavior of the material within the band, formula (1.2) suggest another important conclusion that the rate of the volumetric strain e within the band differs from zero. Thus, the band of localized strain in a porous material is not a pure slip band. When passing through the band, the jump of material rate component, normal to the slip band surface, is nonzero. This has first been observed by Shield in his classical paper on the application of the associated flow law to materials with a Coulomb–Mohr loading surface [3].

Only one conclusion can be drawn from the softening of the material. In the limit $h \to 0$, the localized strain band is the surface of loss of continuity of the material, i.e., the fracture surface. This interpretation agrees with experimental observations (at least for uniaxial or biaxial stretching). Probably, in some cases, violation of the initial hypothesis of the model for small h will be significant, e.g., the mechanisms of defect healing appear immediately in the band. Obviously, in this case, the limiting transition $h \to 0$ is incorrect [3]. We have not studied models of this type.

The most important condition for the appearance of a slip band in the model material studied is the violation of the material's "physical stability," i.e., the Drukker postulate. With our notation this condition is

$$\frac{\partial\Phi}{\partial\theta}\dot{\theta} + \frac{\partial\Phi}{\partial\Gamma}\dot{\Gamma} < 0. \tag{1.3}$$

In the numerical modeling a porous material with an ideally plastic solid phase ($\dot{\Gamma} = 0$) in tension was considered and hence (1.3) was fulfilled.

2. Modeling of the Appearance of Localized Strain Bands. The occurrence of a slip band was modeled within the framework of plane strain for uni- and biaxial stretching of a porous sample with a square cross section. The framework of the porous sample was assumed to be an ideally plastic material. In general, due to increase in porosity, the material softened. The problem of determining the velocity fields for the elements of this material is, generally speaking, incorrect. However, it is physical instability (resulting in

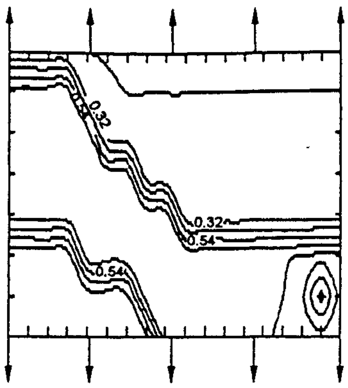

Fig. 1

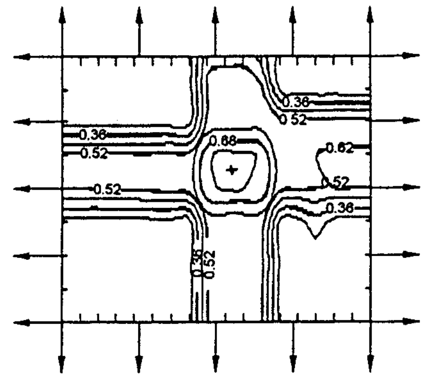

Fig. 2

incorrectness in the statement of the problem) that stimulates the transition of the homogeneous deformation regime to the localized regime. The problem can be solved only by using regularization procedures. In our case, these are the finite-dimensional approximation of the starting equations and the approach in [4] which employs, within the framework of the finite element method, a procedure similar to the averaging of flow rates in the conditional time step used in modeling the process.

Use of the above methods essentially changes the rheological properties of the material because of the appearance of viscosity due to approximation. Generally speaking, the magnitude of this viscosity is unknown. Therefore, it is impossible to determine the evolution of the real process with real time using such a model. The goal of the present paper is to elucidate the geometric characteristics of the deformation regimes that replace homogeneous deformation once a given material loses physical stability. Note that the idea to study the behavior of physically unstable materials is now widely acknowledged [5]. For plastic flow this approach was first employed in [6].

Stretching of a Plane-Deformed Sample with One Inhomogeneity. The position of the localized band in the sample was fixed by fixation of a local inhomogeneity (the regions with greater porosity). The mean porosity of the sample is 0.2, and the porosity of the inhomogeneity region is 0.4. The inhomogeneity region is observed at the lower right of Fig. 1.

In uniaxial stretching a localized strain band forms at an angle to the stretching direction. As the sample is stretched, the porosity in this region increases rapidly to the fracture point of the sample (porosity equal to unity). In the remaining volume of the material the porosity does not change (rigid regions). The axial flow symmetry is broken, i.e., velocities appear that are perpendicular to the stretching direction and correspond to the shear of rigid parts relative to one another. Figure 1 shows the porosity distribution in the sample in a time step.

In the biaxial stretching of the sample, two localized bands arise (Fig. 2). With these deformation schemes, relation (1.1) is violated. Indeed, in this case, a single macroscopic band cannot arise, although localization still proceeds.

Note that the statement that a localized strain band appears in the vicinity of an inhomogeneity would be inaccurate. The inhomogeneity displays a weakened cross section across which the band can pass. The band, in turn, is localized throughout the cross section rather than propagating from the inhomogeneity. This can be readily observed in numerical modeling.

The Interaction of Localized Strain Bands. Upon destruction the real material contains a lot of inhomogeneity centers capable of giving rise to slip bands. The interaction of several localization regions has been studied using the deformation of a sample with two inhomogeneities as an example. In uniaxial stretching each of the inhomogeneities gives rise to a slip band (Fig. 3). The bands are parallel. During deformation the density in the more "powerful" band rapidly decreases and the second band stops to develop. Thus, the material falls into pieces in a single plane.

Quite a different pattern is observed in biaxial deformation. In this case, the interaction between

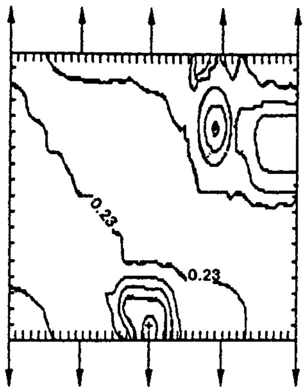

Fig. 3

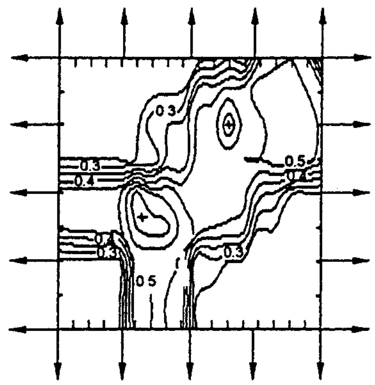

Fig. 4

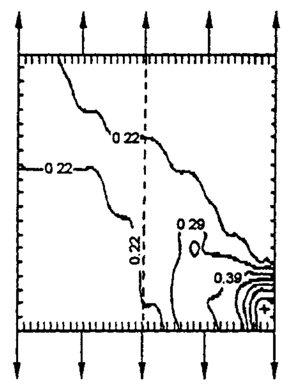

Fig. 5

inhomogeneities has a considerable effect. The localized strain bands join the inhomogeneities into one network (Fig. 4). Destruction occurs simultaneously along all lines of this network.

Influence of the Rheological Properties of Material on the Formation of Localization Regions. The existence of slip bands is usually assigned to the plastic properties of material and this is confirmed by calculations. Even for the nonlinearity exponents $n = 0.1$ in the law of nonlinearly viscous behavior of the material

$$\frac{\sigma}{\sigma_0} = A\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^h$$

the increase in the band with a substantial difference in the porosity within and beyond the band is accompanied by its "spreading." The width of the localization region increases considerably and cannot be considered as localization on rather coarse grids of finite-element patterns.

A similar process (the origin of a slip band followed by its vanishing) is typical of the propagation of the band in a material containing regions of both physically stable and unstable materials. Figure 5 demonstrates the initial stage of deformation. The actual vanishing of the band at the interface can be observed in the region of unstable material. Under the conditions of uniaxial stretching, the physically unstable part cannot be destroyed independently of the stable one. The localization process gradually decays.

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